

Coherent Propagation of Polarized Millimeter Waves Through Falling Hydrometeors

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Abstract—Coherent propagation of polarized millimeter waves through falling nonspherical hydrometeors is considered on the basis of Foldy's approximation. It is assumed that the distribution of hydrometeors over orientations is locally axially symmetric, the axis of symmetry being given by the local direction of air flow around the hydrometeors. An efficient rigorous method is described to compute the orientationally averaged forward-scattering amplitude matrix. This method is based on Waterman's T-matrix approach and fully exploits rotational properties of the T-matrix. First, the T-matrix is analytically averaged over hydrometeor orientations in the local coordinate system with the Z -axis along the direction of local air flow. Then, the elements of the forward-scattering amplitude matrix with respect to the local and laboratory coordinate systems are calculated via simple analytical expressions. Analytical solutions of the propagation equation for the coherent electric field are discussed. Cross polarization of the coherently transmitted linearly polarized wave is computed for canting spheroidal raindrops at 34.8 GHz, and the effects of the scatter of raindrop canting angles on the cross polarization are discussed. In particular, it is shown that the scatter of raindrop canting angles reduces the cross polarization considerably as compared with that of equioriented raindrops.

1. INTRODUCTION

Falling raindrops and other hydrometeors have, in general, nonspherical shape and are given preferred orientations due to gravitational and aerodynamical forces. Furthermore, they exist within a wide spectrum of sizes extending up to a few millimeters. Thus, to calculate coherent propagation of millimeter waves in rain and snow media, one has to consider scattering by partially oriented nonspherical particles with size parameters in the resonance region. First, the orientationally averaged forward-scattering amplitude matrix must be computed at all points of the wave path, and then the corresponding propagation equation for the coherent electric field must be solved to compute the parameters of the transmitted signal (for reviews of the relevant literature, we refer the reader to Oguchi [1] and Olsen [2]).

From general physical considerations, it is obvious that the distribution of falling nonspherical hydrometeors over orientations is always axially symmetric, the axis of symmetry being given by the local direction of air flow around the hydrometeors. The direction of the air flow is equal to the vector addition of the vertical component due to the fall of the particles and the horizontal component

due to the vertical gradient of the wind speed [3]. An additional factor that must be taken into account is the small oscillating component of the horizontal wind velocity [4]. While the mean canting angle is that of Brussaard [3] and is specified by the steady component of the horizontal wind velocity, the small oscillating component is responsible for the angular distribution of hydrometeor orientations about this mean angle.

This physical model of hydrometeor canting together with usual assumption of rotationally symmetric hydrometeor shape dictate the use of the following three coordinate systems to calculate the coherent propagation of electromagnetic waves. The first one, hereafter referred to as the laboratory coordinate system, is used to specify the experimental conditions and is chosen such that the Z -axis coincides with the vertical direction. The second one, hereafter referred to as the local coordinate system, is chosen such that the Z -axis is coincident with the direction of the local air flow around the hydrometeors. Thus, the Z -axis of the local coordinate system is the axis of symmetry of the local orientation distribution of the hydrometeors. The third one, hereafter referred to as the body frame, is fixedly attached to the rotationally symmetric hydrometeor and is chosen such that the Z -axis coincides with the axis of particle symmetry. Thus, orientation of the hydrometeor with respect to the local coordinate system may be specified by the Eulerian angles of rotation (α, β, γ) [5, 6], and the orientation distribution of the hydrometeors is given by a function of only one Eulerian angle β .

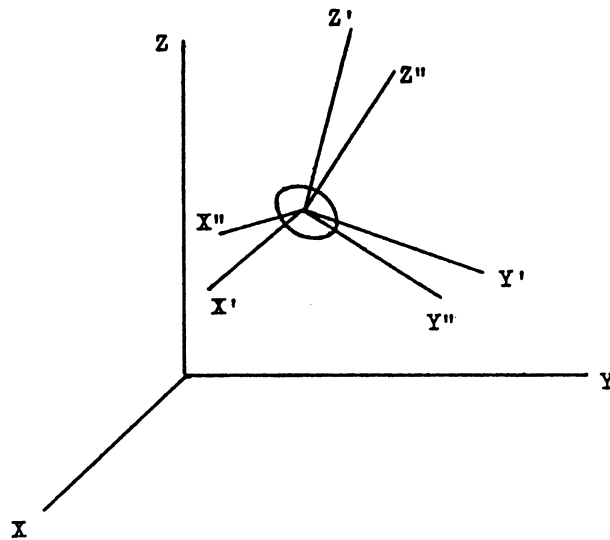


Figure 1. Laboratory (XYZ), local ($X'Y'Z'$), and body ($X''Y''Z''$) reference frames. The axis Z coincides with the vertical direction. The axis Z' coincides with the direction of the local air flow around the hydrometeors. The axis Z'' coincides with the axis of hydrometeor symmetry.

The body frame is ideally suited to compute single scattering on a rotationally symmetric particle by using Waterman's T-matrix approach [7]. Then, the local coordinate system is very convenient in computing the orientationally averaged T-matrix, because the orientation distribution is axially symmetric, and rotational properties of the T-matrix can fully be taken into account [6]. After computing the orientationally averaged T-matrix, the forward-scattering amplitude matrix can easily be computed for any angle between the Z -axis of the local coordinate system and the direction of wave propagation. This forward-scattering amplitude matrix, computed with respect to the local coordinate system, can easily be recomputed with respect to the laboratory coordinate system. Finally, this laboratory coordinate system may be used to solve the propagation equation for the coherent electric field.

In this paper, we describe an efficient rigorous procedure to calculate the coherent propagation of polarized millimeter waves through partially aligned, nonspherical, resonance hydrometeors. In Section 2, an analytical method for computing the orientationally averaged forward-scattering amplitude matrix is outlined. This method is based on the T-matrix approach and fully exploits rotational properties of the T-matrix, as derived in [6]. First, the T-matrix is analytically averaged over hydrometeor orientations in the local coordinate system, and then this orientationally averaged T-matrix is used to compute the elements of the forward-scattering amplitude matrix with respect to the local and laboratory coordinate systems via simple analytical expressions. In Section 3, analytical solutions of the propagation equation are briefly discussed. In Section 4, parameters of the transmitted linearly polarized wave are computed for a number of scattering models, and the effects of the scatter of raindrop canting angles on the cross polarization are discussed.

We note here that the problem considered is mathematically identical to the problem of computing interstellar extinction and polarization due to nonspherical dust grains partially aligned in cosmic magnetic fields. That astrophysical problem was discussed in detail in our recent paper [8], and the results of that publication will be used in this study.

2. CALCULATION OF THE ORIENTATIONALLY AVERAGED FORWARD-SCATTERING AMPLITUDE MATRIX

In this paper, we exploit definitions and notations of Ishimaru and Yeh [9]. To calculate the parameters of the coherently transmitted wave one has to solve (analytically or numerically) the propagation equation

$$\frac{d}{ds}[E_c] = [M][E_c] \quad (1)$$

(the factor $\exp[iks - i\omega t]$ is omitted). Here, $[E_c] = \begin{bmatrix} \langle E_v \rangle \\ \langle E_h \rangle \end{bmatrix}$ is the coherent electric field propagating in the direction \hat{s} (subscripts v and h label the vertical and horizontal components, respectively) the pathlength element ds is measured along \hat{s} , and angle brackets denote the ensemble average. The 2×2 matrix $[M]$

is given by

$$[M] = i 2\pi \rho k^{-1} \langle [f(\hat{s}, \hat{s})] \rangle \quad (2)$$

where $[f(\hat{s}, \hat{s})]$ is the forward-scattering amplitude matrix, k is the free-space wavenumber, and ρ is the number of particles per unit volume.

Differential equation (1) may be solved analytically or by using well-known computational procedures, which usually presents no difficulties. Thus the principal problem is to calculate the elements of the ensemble-averaged forward-scattering amplitude matrix. Although calculation of the ensemble-averaged quantities implies, in general, integration over the size, shape, refractive index, and orientation distributions, in this paper we shall discuss only the orientational averaging. All the other averages can be computed by using straightforward numerical integrations. In Section 2.A, we calculate the orientationally averaged forward-scattering amplitude matrix with respect to the local coordinate system. In Section 2.B, transition to the laboratory coordinate system is considered.

A. Local Coordinate System

For calculating the elements of the amplitude scattering matrix $f^{\text{loc}}(\hat{s}, \hat{s}')$ for a nonspherical particle in a fixed orientation with respect to the local coordinate system, we use the T-matrix approach as outlined by Tsang et al. [6]. The notations and definitions used here are those adopted in [8, 10] and slightly differ from those of Tsang et al. In terms of the T-matrix elements, the amplitude scattering matrix is expressed as [6, 7]

$$\begin{aligned} [f^{\text{loc}}(\theta, \phi; \theta', \phi')] &= \frac{4\pi}{k} \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m=-n}^n \sum_{m'=-n'}^{n'} i^{n'-n-1} (-1)^{m-m'} \\ &\times d_n d_{n'} \exp[i(m\phi - m'\phi')] \\ &\times \left\{ [T_{mnm'n'}^{11} \bar{C}_{mn}(\theta) + T_{mnm'n'}^{21} i \bar{B}_{mn}(\theta)] \bar{C}_{m'n'}^*(\theta') \right. \\ &\left. + [T_{mnm'n'}^{12} \bar{C}_{mn}(\theta) + T_{mnm'n'}^{22} i \bar{B}_{mn}(\theta)] \bar{B}_{m'n'}^*(\theta')/i \right\} \quad (3) \end{aligned}$$

where dyadic notations are used, the asterisk denotes the conjugate complex value, $T_{mnm'n'}^{pq}$ are elements of the T-matrix calculated with respect to the local coordinate system,

$$\bar{B}_{mn}(\theta) = \hat{\theta} \frac{d}{d\theta} d_{0m}^n(\theta) + \hat{\phi} \frac{im}{\sin \theta} d_{0m}^n(\theta) \quad (4)$$

$$\bar{C}_{mn}(\theta) = \hat{\theta} \frac{im}{\sin \theta} d_{0m}^n(\theta) - \hat{\phi} \frac{d}{d\theta} d_{0m}^n(\theta) \quad (5)$$

$$d_n = \left[\frac{2n+1}{4\pi n(n+1)} \right]^{1/2} \quad (6)$$

and $d_{mm'}^n(\theta)$ are Wigner d -functions [5]. To use (3) to calculate the elements of the orientationally averaged forward-scattering amplitude matrix, we are to average the T-matrix over all hydrometeor orientations. Let (α, β, γ) be the Eulerian angles of rotation that transform the local coordinate system into the body frame of a hydrometeor [5]. Since orientation distribution of hydrometeors is axially symmetric with respect to the Z -axis of the local coordinate system, we have [8]

$$\langle T_{mnm'n'}^{pq} \rangle = \frac{1}{4\pi^2} \int_0^{2\pi} d\alpha \int_0^\pi d\beta \sin \beta p(\beta) \int_0^{2\pi} d\gamma T_{mnm'n'}^{pq} \quad (7)$$

with [6]

$$T_{mnm'n'}^{pq} = \sum_{m_1=-M}^M D_{m_1m'}^{-1n'}(\alpha, \beta, \gamma) \hat{T}_{m_1nm_1n'}^{pq} D_{mm_1}^n(\alpha, \beta, \gamma) \quad (8)$$

where $\frac{1}{4\pi^2} p(\beta)$ is a probability density function normalized as

$$\int_0^\pi d\beta \sin \beta p(\beta) = 1 \quad (9)$$

$D_{mm'}^n(\alpha, \beta, \gamma)$ are Wigner D -functions given by [5]

$$D_{mm'}^n(\alpha, \beta, \gamma) = \exp(-im\alpha) d_{mm'}^n(\beta) \exp(-im'\gamma) \quad (10)$$

$\hat{T}_{m_1nm_1n'}^{pq}$ are elements of the T-matrix of the rotationally symmetric hydrometeor calculated with respect to the body frame [6], and $M = \min(n, n')$. By use of the Clebsch-Gordan expansion [5]

$$d_{mm_1}^n(\beta) d_{m'm_1}^{n'}(\beta) = (-1)^{m+m_1} \sum_{n_1=|n-n'|}^{n+n'} \langle nm n' - m | n_1 0 \rangle d_{00}^{n_1}(\beta) \times \langle nm_1 n' - m_1 | n_1 0 \rangle \quad (11)$$

where $\langle nm n' m' | n_1 m_1 \rangle = C_{nm n' m'}^{n_1 m_1}$ are Clebsch-Gordan coefficients, and taking into account the symmetry properties

$$\hat{T}_{-mn-mn'}^{pq} = (-1)^{p+q} \hat{T}_{mnmn'}^{pq} \quad (12)$$

and

$$\langle n - mn' m | n_1 0 \rangle = (-1)^{n+n'+n_1} \langle nm n' - m | n_1 0 \rangle \quad (13)$$

we easily derive from the above formulae [8]

$$\langle T_{mnm'n'}^{pq} \rangle = \delta_{mm'} T_{mnn'}^{pq} \quad (14)$$

with

$$T_{mnn'}^{pq} = (-1)^m \sum_{n_1=|n-n'|}^{n+n'} \left[1 + (-1)^{n+n'+n_1+p+q} \right] \langle nm n' - m | n_1 0 \rangle \times p_{n_1} \sum_{m_1=0}^M (-1)^{m_1} \left[1 - \frac{1}{2} \delta_{m_1 0} \right] \langle nm_1 n' - m_1 | n_1 0 \rangle \hat{T}_{m_1nm_1n'}^{pq} \quad (15)$$

where δ_{mn} is the Kronecker delta and

$$p_n = \int_0^\pi d\beta \sin \beta p(\beta) d_{00}^n(\beta) \quad (16)$$

In other words, the quantities p_n are coefficients in the expansion of the function $p(\beta)$ in Legendre polynomials $P_n(\cos \beta) = d_{00}^n(\beta)$:

$$p(\beta) = \sum_{n=0}^{\infty} \frac{2n+1}{2} p_n P_n(\cos \beta) \quad (17)$$

From (12), (13), and (15), one has

$$T_{-mnn'}^{pq} = (-1)^{p+q} T_{mnn'}^{pq} \quad (18)$$

By using (3)–(6), (14), (18), and the symmetry relation

$$d_{-m-m'}^n(\theta) = (-1)^{m+m'} d_{mm'}^n(\theta) \quad (19)$$

we derive for the elements of the orientationally averaged forward-scattering amplitude matrix with respect to the local coordinate system

$$\begin{aligned} \langle f_{vv}^{\text{loc}}(\hat{s}, \hat{s}) \rangle &= \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m=0}^M (2 - \delta_{m0}) \beta_{nn'} \\ &\times \left[T_{mnn'}^{11} \frac{m^2}{\sin^2 \theta} d_{0m}^n(\theta) d_{0m}^{n'}(\theta) + T_{mnn'}^{12} \frac{m}{\sin \theta} d_{0m}^n(\theta) \frac{d}{d\theta} d_{0m}^{n'}(\theta) \right. \\ &\left. + T_{mnn'}^{21} \frac{d}{d\theta} d_{0m}^n(\theta) \frac{m}{\sin \theta} d_{0m}^{n'}(\theta) + T_{mnn'}^{22} \frac{d}{d\theta} d_{0m}^n(\theta) \frac{d}{d\theta} d_{0m}^{n'}(\theta) \right] \quad (20) \end{aligned}$$

$$\langle f_{vh}^{\text{loc}}(\hat{s}, \hat{s}) \rangle = \langle f_{hv}^{\text{loc}}(\hat{s}, \hat{s}) \rangle = 0 \quad (21)$$

$$\begin{aligned} \langle f_{hh}^{\text{loc}}(\hat{s}, \hat{s}) \rangle &= \sum_{n=1}^{\infty} \sum_{n'=1}^{\infty} \sum_{m=0}^M (2 - \delta_{0m}) \beta_{nn'} \\ &\times \left[T_{mnn'}^{11} \frac{d}{d\theta} d_{0m}^n(\theta) \frac{d}{d\theta} d_{0m}^{n'}(\theta) + T_{mnn'}^{12} \frac{d}{d\theta} d_{0m}^n(\theta) \frac{m}{\sin \theta} d_{0m}^{n'}(\theta) \right. \\ &\left. + T_{mnn'}^{21} \frac{m}{\sin \theta} d_{0m}^n(\theta) \frac{d}{d\theta} d_{0m}^{n'}(\theta) + T_{mnn'}^{22} \frac{m^2}{\sin^2 \theta} d_{0m}^n(\theta) d_{0m}^{n'}(\theta) \right] \quad (22) \end{aligned}$$

where

$$\beta_{nn'} = k^{-1} i^{n'-n-1} \left[\frac{(2n+1)(2n'+1)}{n(n+1)n'(n'+1)} \right]^{1/2} \quad (23)$$

Note that, due to the axial symmetry, the matrix $[\langle f^{\text{loc}}(\hat{s}, \hat{s}) \rangle]$ does not depend on the azimuthal angle ϕ .

Thus, to calculate the orientationally averaged forward-scattering amplitude matrix with respect to the local coordinate system $\langle [f^{\text{loc}}(\hat{s}, \hat{s})] \rangle$ one may use (15) and (20)–(22).

Formulae for computing the matrix $[\hat{T}]$ for rotationally symmetric homogeneous particles are given in Appendix C of Tsang et al. [6]. Numerical aspects of the T-matrix computations are discussed in detail in [11]. Convenient formulae for computing the Clebsch-Gordan coefficients, appearing in (15), and the angular functions, appearing in (20) and (22), are given in Appendices A and B or [8].

The use of the above formulae is computationally very efficient. As was shown in [8], calculation of the orientationally averaged T-matrix and the forward-scattering amplitude matrix $\langle [f^{\text{loc}}(\hat{s}, \hat{s})] \rangle$ requires only a small fraction of the time that is necessary for calculating the particle T-matrix with respect to the body frame, i.e., the matrix $[\hat{T}]$. In other words, computations for axially symmetric orientation distributions of nonspherical hydrometeors require practically the same computer time as computations for equioriented hydrometeors.

B. Transition to the Laboratory Coordinate System

Denote by (\hat{s}, Z) the plane through the vector \hat{s} and the Z -axis of the laboratory coordinate system and by $(\hat{s}, Z_{\text{loc}})$ the plane through the vector \hat{s} and the Z -axis of the local coordinate system. Assume that the orientationally averaged forward-scattering amplitude matrix $\langle [f^{\text{loc}}(\hat{s}, \hat{s})] \rangle$ is known. Let Ω be the angle of the rotation of the plane (\hat{s}, Z) around the vector \hat{s} that transforms this plane into the plane $(\hat{s}, Z_{\text{loc}})$. The angle Ω is measured in the clockwise direction, when looking in the direction of wave propagation. Then we have for the forward-scattering amplitude matrix with respect to the laboratory coordinate system

$$\langle [f(\hat{s}, \hat{s})] \rangle = [G(-\Omega)] \langle [f^{\text{loc}}(\hat{s}, \hat{s})] \rangle [G(\Omega)] \quad (24)$$

where

$$[G(\Omega)] = \begin{bmatrix} \cos \Omega & \sin \Omega \\ -\sin \Omega & \cos \Omega \end{bmatrix} \quad (25)$$

3. ANALYTICAL SOLUTIONS OF THE PROPAGATION EQUATION

General solution of (1) for arbitrary inhomogeneous media is given by

$$[E_c(s)] = [T(s)] [E_c(0)] \quad (26)$$

Here, $[T(s)]$ is a 2×2 matrix which satisfies the initial condition

$$[T(0)] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (27)$$

Analytically, the matrix $[T(s)]$ may be written as a matricant [12]. Specifically, let us divide the interval $[0, s]$ into N equal subintervals $[s_{n-1}, s_n]$, $n = \overline{1, N}$, with $s_0 = 0, s_N = s$ and $s_n - s_{n-1} = \Delta s = s/N$. Then

$$\begin{aligned} [T(s)] &= \lim_{N \rightarrow \infty} \left\{ [G(-\Omega_N)] \exp \left[\begin{bmatrix} M^{\text{loc}} \\ \Delta s \end{bmatrix}_N \right] [G(\Omega_N)] \right. \\ &\quad \times [G(-\Omega_{N-1})] \exp \left[\begin{bmatrix} M^{\text{loc}} \\ \Delta s \end{bmatrix}_{N-1} \right] [G(\Omega_{N-1})] \end{aligned}$$

$\times \dots$

$$\times [G(-\Omega_1)] \exp \left[\left[M^{\text{loc}} \right]_1 \Delta s \right] [G(\Omega_1)] \} \quad (28)$$

where the angles of rotation are defined in Section 2.B. Since the matrices $[M^{\text{loc}}]_n$ for locally axially symmetric media are diagonal, we have

$$\exp \left[\left[M^{\text{loc}} \right]_n \Delta s \right] = \begin{bmatrix} \exp \left[(M_{vv}^{\text{loc}})_n \Delta s \right] & 0 \\ 0 & \exp \left[(M_{hh}^{\text{loc}})_n \Delta s \right] \end{bmatrix} \quad (29)$$

For homogeneous media, we have from the above equations

$$[T(s)] = [G(-\Omega)] \exp \left\{ \left[M^{\text{loc}} \right] s \right\} [G(\Omega)] \quad (30)$$

with

$$T_{vv}(s) = \cos^2 \Omega \exp(M_{vv}^{\text{loc}} s) + \sin^2 \Omega \exp(M_{hh}^{\text{loc}} s) \quad (31)$$

$$T_{vh}(s) = T_{hv}(s) = \cos \Omega \sin \Omega \left[\exp(M_{vv}^{\text{loc}} s) - \exp(M_{hh}^{\text{loc}} s) \right] \quad (32)$$

$$T_{hh}(s) = \sin^2 \Omega \exp(M_{vv}^{\text{loc}} s) + \cos^2 \Omega \exp(M_{hh}^{\text{loc}} s) \quad (33)$$

4. NUMERICAL RESULTS

In this section, we report illustrative numerical results for monodisperse oblate spheroidal raindrops at 34.8 GHz. The ratio of the semi-axes of the spheroid is 1.6, the radius of the equal-volume sphere is 3.5 mm, and the refractive index is $5.048 + 2.794i$. We calculate the cross-polar discrimination factors XPD_V and XPD_H given by

$$XPD_V(s) = 20 \log |T_{vh}(s) / T_{vv}(s)| \quad (34)$$

and

$$XPD_H(s) = 20 \log |T_{vh}(s) / T_{hh}(s)| \quad (35)$$

The scattering medium is homogeneous, and the elements of the matrix $[T(s)]$ are computed via (31)–(33). The propagation path is horizontal, and the axis of the orientation distribution of the raindrops is perpendicular to the direction of wave propagation and is Ω degrees from the vertical with $\Omega = 5^\circ$ and 10° . To model partial alignment of the raindrops, we use the simplest distribution function

$$p(\beta) = \frac{1}{2} + P_2(\cos \beta) = \frac{3}{2} \cos^2 \beta \quad (36)$$

The number density ρ of the monodisperse raindrops is set at an arbitrary value of 10 particles/m³.

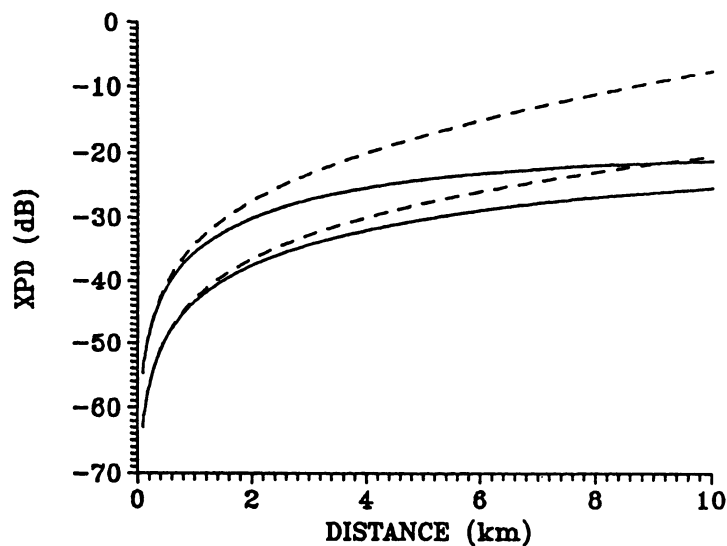


Figure 2. Cross-polar discriminations XPD_V (solid lines) and XPD_H (dashed lines) versus distance for canting spheroidal raindrops with $\Omega = 5^\circ$. The lower curves are for partially aligned raindrops, and the upper curves are for equioriented raindrops.

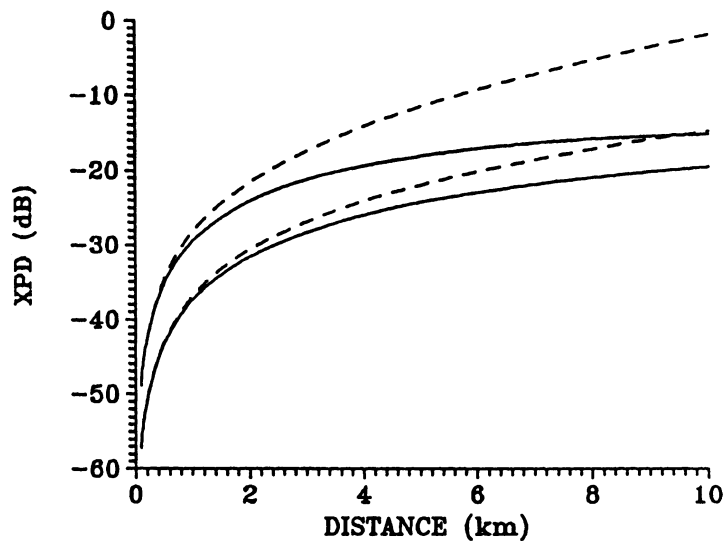


Figure 3. Cross-polar discriminations XPD_V (solid lines) and XPD_H (dashed lines) versus distance for canting spheroidal raindrops with $\Omega = 10^\circ$. The lower curves are for partially aligned raindrops, and the upper curves are for equioriented raindrops.

Figures 2 and 3 show the relation between the cross polarization and propagation path length for partially aligned raindrops. For comparison, analogous computations are shown for equioriented raindrops. From these figures, the following obvious conclusions can be made.

- (a) At small mean canting angles Ω , the cross polarization increases with increase of Ω , XPD_V being smaller than XPD_H .
- (b) Difference between XPD_V and XPD_H for partially aligned hydrometeors is smaller than that for equioriented raindrops.
- (c) At large distances, XPD_V saturates (much) faster than XPD_H for both partially aligned and equioriented hydrometeors (cf. Oguchi [1]).
- (d) The scatter of the raindrop canting angles reduces the cross polarization considerably as compared with that for equioriented raindrops. The same conclusion was made earlier by Oguchi [13] who used a simplified model of raindrop orientation distribution.

5. CONCLUSIONS

The problem of coherent transmission of polarized millimeter waves through falling nonspherical hydrometeors was studied. A realistic physical model of hydrometeor canting was used, according to which the distribution of falling hydrometeors over orientations is locally axially symmetric, the axis of symmetry being given by the local direction of air flow around the hydrometeors. Waterman's T-matrix approach was used to calculate analytically the orientationally averaged forward-scattering amplitude matrix. The propagation equation for the coherent electric field was solved to compute the cross polarization or the transmitted wave. The computations were performed for partially aligned and equioriented spheroidal raindrops at 34.8 GHz. These computations evidently shown that possible scatter of hydrometeor canting angles can significantly affect the cross polarization of the transmitted signal and must be taken into account in interpreting the experimental data.

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